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AUTHOR(S) **Peter Herczeg, T-5**

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PION AND MUON DECAYS BEYOND THE STANDARD MODEL

Peter Herczeg

Theoretical Division
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

ABSTRACT

We review and discuss the information provided by charged pion and muon decays on physics beyond the minimal standard model.

1. INTRODUCTION

Pion and muon decays are among the oldest tools of particle physics. Their study helped to develop the standard model. Today their main role is to probe for possible deviations from the predictions of the minimal standard model. The minimal standard model has been spectacularly successful in accounting for the existing data. Nevertheless, for many theoretical reasons the presence of new physics is expected.

In this talk we shall review and discuss what type of new physics can be probed in pion and muon decays, what we have learned so far, and what further experiments would be worthwhile. From pion decays we shall discuss here only the decays of the charged pion; for reviews of neutral pion decays we refer the reader to Ref. 1.

In Section 2 we discuss muon decays involving neutrinos in the final state, such as the usual muon decay. In Section 3 we shall consider the decays of the $\pi^{\pm}$. Charged pion decays probe some new aspects of the leptonic interactions, and also new interactions that do not contribute to leptonic processes in lowest order. Section 4 deals with lepton family number violating decays, such as $\mu \to e\gamma$. Here we include also a discussion of muonium $\to$ antimuonium conversion. In Section 5 we consider decay modes involving some possible new particles. The last section contains our conclusions.

2. MUON DECAYS INTO NEUTRINOS

2.1. $\mu^+ \to e^+$ +neutrinos

In the minimal standard model[2] the main decay mode of the muon (μ^+ for definiteness) is due to W-exchange and proceeds as

$$\mu^+ \to e^+ \; \nu_e \bar{\nu}_\mu \tag{1}$$

where ν_e and ν_μ are massless left-handed neutrinos accompanying, respectively, the left-handed electron and the left-handed muon in doublet representations of $SU(2)_L$. The effective interaction describing (1) is the V-A Hamiltonian[3]

$$H^{(\mu)}_{V-A} \; = \; (G_F/\sqrt{2}) \; \bar{\mu}\gamma_\lambda \, (1-\gamma_5)\nu_\mu \; \bar{\nu}_e \gamma^\lambda \, (1-\gamma_5) e \; + \; H.c. \tag{2}$$

where $G_F \; = \; (g^2/8m^2_W)(1+\Delta r)$; Δr are radiative corrections[4]. In the following we shall refer to the process $\mu^+ \to e^+ \; + \; neutrinos$ simply as "muon-decay."[5] In extensions of the minimal standard model the interaction responsible for muon-decay may include new components (for example new V,A interactions, or contributions from S,P,T couplings). The neutrino states emitted in muon-decay may also be more involved: the neutrinos may be massive fermions (Dirac or Majorana), and the neutrino gauge-group eigenstates may be not particular mass-eigenstates, but linear combinations of them.

Let us consider the decay $\mu^+ \to e^+ \nu_e \nu_\mu$ in a framework where the muon-decay interaction is described by the most general local nonderivative lepton-family-number[6] conserving four-fermion interaction[7], and the neutrinos are mass-eigenstates, with masses small enough that their effect on the positron spectrum can be neglected. In the helicity projection form[8,9] the corresponding Hamiltonian can be written as

$$\begin{aligned} H^{(\mu)} \; = & \sum_{\alpha,\beta = L,R} (g^V_{\alpha\beta} \; \bar{e}\gamma_\lambda \Gamma_\alpha \nu_e \bar{\nu}_\mu \gamma^\lambda \Gamma_\beta \mu \; + \; g^S_{\alpha\beta} \; \bar{e}\Gamma'_\alpha \nu_e \bar{\nu}_\mu \Gamma_\beta \mu) \\ & + \sum_{\substack{\alpha,\beta = L,R \\ \alpha \neq \beta}} (g^T_{\alpha\beta} \; \bar{e} t_{\rho\sigma} \Gamma'_\alpha \nu_e \bar{\nu}_\mu t_{\rho\sigma} \Gamma_\beta \mu) \; + \; H.c., \end{aligned} \tag{3}$$

where $t_{\rho\sigma} = (i/2\sqrt{2})(\gamma_\rho\gamma_\sigma - \gamma_\sigma\gamma_\rho), \Gamma'_R = \Gamma_L = (1 - \gamma_5)$, and $\Gamma'_L = \Gamma_R = (1 + \gamma_5)$. The subscripts α and β on the coupling constants indicate the handedness of the electron and of the muon, respectively. The Hamiltonian (3) contains 19 real parameters (10 complex coupling constants minus an overall phase). The minimal standard model Hamiltonian (2) is obtained from (3) by setting $g^V_{LL} = G_F/\sqrt{2}$, and all the other coupling constants equal to zero. If the neutrinos are not detected, only ten quadratic functions of the coupling constants (denoted in the literature by a,a',b,b',c,c',α, α', β and β') can be determined[10]. Experimental constraints on the coupling constants in (3) have been analyzed recently in Ref. 11. The muon-lifetime τ_μ, the positron energy spectrum parameters ρ, δ, ξ, and the positron longitudinal polarization parameters ξ', ξ'' determine the six constants a,a',b,b',c and c', or equivalently, the following six independent positive semidefinite quadratic forms of the coupling constants that can be formed from them[11]:

$$
\begin{aligned}
G_\mu^2 = {} & \frac{1}{2}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2\right) \\
& + 2\left(|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{RL}|^2 + |g^V_{LL}|^2\right) \\
& + 6\left(|g^T_{RL}|^2 + |g^T_{LR}|^2\right)
\end{aligned}
\tag{4}
$$

$$Q_{RR} = \left(\frac{1}{4}\left|g^S_{RR}\right|^2 + \left|g^V_{RR}\right|^2\right)\left(G_\mu/\sqrt{2}\right)^{-2}, \tag{5}$$

$$Q_{LR} = \left(\frac{1}{4}\left|g^S_{LR}\right|^2 + \left|g^V_{LR}\right|^2 + 3\left|g^T_{LR}\right|^2\right)\left(G_\mu/\sqrt{2}\right)^{-2}, \tag{6}$$

$$Q_{RL} = \left(\frac{1}{4}\left|g^S_{RL}\right|^2 + \left|g^V_{RL}\right|^2 + 3\left|g^T_{LR}\right|^2\right)\left(G_\mu/\sqrt{2}\right)^{-2}, \tag{7}$$

$$B_{LR} = \left(\frac{1}{16}\left|g^S_{LR} + 6g^T_{LR}\right|^2 + \left|g^V_{LR}\right|^2\right)\left(G_\mu/\sqrt{2}\right)^{-2}, \tag{8}$$

and

$$B_{RL} = \left(\frac{1}{16}\left|g^S_{RL} + 6g^T_{RL}\right|^2 + \left|g^V_{RL}\right|^2\right)\left(G_\mu/\sqrt{2}\right)^{-2}. \tag{9}$$

The constant G_μ is determined by the decay rate (neglecting radiative corrections $G_\mu^2 = 192\pi^3\Gamma_\mu/m_\mu^5$; with radiative corrections included $G_\mu = 1.16637(2) \times 10^{-5}$ GeV^{-2} (Ref. 12)). The present experimental values of the muon-decay parameters are summarized in Table I. They imply values for

$Q_{RR}, Q_{RL}, Q_{LR}, B_{LR}$ and B_{RL} consistent with zero[11]. The resulting limits on the coupling constants (taken from the updated fit given in Ref. 13) are shown in Table II. The only combination of the coupling constants which is different from zero is the quantity Q_{LL} defined by

$$Q_{LL} \equiv 1 - Q_{RR} - Q_{LR} - Q_{RL} = \left(\frac{1}{4}\left|g^S_{LL}\right|^2 + \left|g^V_{LL}\right|^2\right)(G_\mu/\sqrt{2})^{-2}, \quad (10)$$

with a lower bound[19]

$$Q_{LL} > 0.949 \qquad (90\% \text{ c.l.}). \quad (11)$$

To obtain limits on the individual constants g^S_{LL} and g^V_{LL} in (10) one needs additional information. This can be obtained from the results of measurements of the inverse muon-decay process $\nu_\pi e^- \to \mu^- n_i$ (Ref. 11), where ν_π is the neutrino state emitted in $\pi^+ \to \mu^+ \nu_\pi$ decay, and n_i are some neutrino states. The total cross section S for $(\nu_\pi e^- \to \mu^- n_1) + (\nu_\pi e^- \to \mu^- n_2) + \ldots$ relative to the cross-section predicted by the minimal standard model is found to be[20]

$$S = \left(\sum_i \sigma(\nu_\pi e^- \to \mu^- n_i) / \sigma(\nu_\mu e^- \to \mu^- \nu_e)_{V-A}\right) = 0.98 \pm 0.12 \quad . \quad (12)$$

Assuming that $\nu_\pi = \nu_\mu$ (Ref. 21) only ν_e contributes in our framework to the sum in (12), and one can use the Hamiltonian (3) to calculate S. Integrating over the neutrino spectrum in the experiment of Ref. 20, the result for S (taking into account the constraints on the quantities (5) - (9)) is[11]

$$S \simeq \left[-h|g^V_{LL}|^2 + (3/64)(1+h)|g^S_{LL}|^2\right](G_\mu/\sqrt{2})^{-2}, \quad (13)$$

where h is the longitudinal polarization of ν_μ in $\pi^+ \to \mu^+\nu_\mu$ decay. From the experimental lower bound $P_\mu \xi \delta/\rho > 0.99682$ (see Table I) one can deduce[22]

$$1 + h < 0.00318 \qquad (90\% \text{ c.l.}). \quad (14)$$

Consequently (since the experimental muon-decay rate requires $|g^S_{LL}|^2 < 4(G_\mu/\sqrt{2})^2$), the second term in (13) can be neglected. One has therefore

$$S \simeq |g^V_{LL}|^2 (G_\mu/\sqrt{2})^{-2} \quad . \quad (15)$$

Eq. (15) and the experimental result (12) lead to

$$|g^V_{LL}| > 0.888\ (G_\mu/\sqrt{2}) \qquad (90\%\ \text{c.l.}). \qquad (16)$$

as $Q_{LL} \leq 1$, one obtains also[11]

$$|g^S_{LL}| < 0.918\ (G_\mu/\sqrt{2}) \qquad (90\%\ \text{c.l.}). \qquad (17)$$

The conclusion is that given our assumptions regarding the muon-decay interaction and the nature of the neutrinos involved, the only term in the Hamiltonian (3) which we know to be nonzero is the one present in the minimal standard model. Moreover, this term is responsible for at least 79% of the observed muon-decay rate[13]. At the same time, some of the non - (V-A) interactions could still have a strength comparable or not too much weaker than G_μ.

The parameters α, β, α' and β' determine the transverse components of the positron polarization. In the minimal standard model $\alpha = \beta = \alpha' = \beta' = 0$. Let us consider α' and β', which (neglecting higher order effects) are responsible for the time-reversal-odd correlation $\vec{\sigma}_\mu \cdot \vec{p}_e \times \vec{\sigma}_e$ (the only one that can be formed if the neutrinos are unobserved). They are given by [9]

$$\alpha'/A = Im\,[g^V_{LR}\,(g^{S*}_{RL} + 6g^{T*}_{RL}) - g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR})]/G_\mu{}^2 \quad , \qquad (18)$$

$$\beta'/A = -\frac{1}{2}\,Im\,(g^V_{LL}\,g^{S*}_{RR} - g^V_{RR}\,g^{S*}_{LL})/G^2_\mu \quad , \qquad (19)$$

where $A = 8G^2_\mu$.

The parameter α' is nonzero only if there is an interference between two non-standard contributions and is therefore less sensitive to the new couplings it depends on than β' is to g^S_{RR}. A g^S_{RR}-term added to the V-A contribution would yield

$$\alpha'/A = 0 \quad , \qquad (20)$$

$$\beta'/A = (2\sqrt{2}\,G_\mu)^{-1}\,Im\,g^S_{RR} \quad . \qquad (21)$$

The direct measurement of β'/A gives (cf. Table I)

$$|Im\,g^S_{RR}| < 0.132\ G_\mu \qquad (90\%\ \text{c.l.}). \qquad (22)$$

A better limit is provided at present by the bound on $|g^S_{RR}|$ (cf. Table II). This yields

$$|Im\ g^S_{RR}| < 0.047 G_\mu \quad . \tag{23}$$

It should be noted that if the neutrinos are Majorana particles, then in the presence of neutrino mixing the correlation $\vec{\sigma}_\mu \cdot \vec{p}_e \times \vec{\sigma}_e$ could arise even if there are no other than V,A fundamental currents.[23]

What kind of new physics could lead to the non-standard terms in the Hamiltonian (3)?

S, P couplings could arise at the tree level through the exchange of charged Higgs bosons ($H^\pm$).[24] Charged Higgs bosons are present e.g. if the Higgs sector of the standard model contains more than one Higgs doublet. In models where the charged Higgs-fermion couplings are proportional to the masses of the fermions involved, the scalar couplings are too small to be observable even for relatively light $H^\pm$. E.g. for $g^S_{\alpha\beta} \simeq m_\mu m_e G_F/M_H^2$ and $M_H \simeq 3$ GeV one would have $g^S_{\alpha\beta} \simeq 6 \times 10^{-6} G_F$. In some models the Higgs-fermion couplings are proportional to the masses of some heavy fermions (f) in the theory.[25] Then $g^S_{\alpha\beta} \sim m_f^2 G_F/M_H^2$ so that $g^S_{\alpha\beta}$ could be as large as the limits in Table II.

New V,A interactions are present for example in left-right symmetric models based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$.[26] The muon-decay interaction in the absence of lepton mixing is of the form (3) with

$$g^V_{LL} \simeq g_L^2 \cos^2\zeta/8m_1^2 \quad , \tag{24}$$

$$g^V_{RR}/g^V_{LL} \simeq g_R^2 m_1^2/g_L^2 m_2^2 \quad , \tag{25}$$

$$g^V_{LR}/g^V_{LL} = g^{V*}_{RL}/g_{LL} \simeq -(g_R/g_L)\,\zeta e^{i\omega} \quad , \tag{26}$$

where g_L and g_R are the gauge coupling constants corresponding to $SU(2)_L$ and $SU(2)_R$, m_1 and m_2 are the masses of the two charged gauge bosons W_1 and W_2, ζ is the $W_L - W_R$ mixing angle, and ω is a CP-violating phase.

The best limits on $|g^V_{\alpha\beta}/g^V_{LL}|$ ($\alpha\beta = LR, RL, RR$) from muon decay are

$$|g^V_{LR}/g^V_{LL}| = |g^V_{RL}/g^V_{LL}| < 0.033 \tag{27}$$

(from the ρ-parameter), and

$$|g^V_{RR}/g^V_{LL}| < 0.04 \tag{28}$$

(from the lower bound on $P_\mu\xi\delta/\rho$).

For $g_R = g_L$ and m_1 =81 MeV (28) implies $m_2 > 70$ GeV. In manifestly left-right symmetric models ($g_R = g_L$, equal left- and right-handed quark mixing matrices) the upper limits on $|g^V_{LR}/g^V_{LL}|$, $|g^V_{RL}/g^V_{LL}|$ and g^V_{RR}/g^V_{LL} from nonleptonic transitions are smaller by about an order of magnitude (but not as reliable). For non-manifestly left-right symmetric models (27) and (28) are the best available limits on $(g_R/g_L)|\zeta|$ and $g_R^2 m_1^2/g_L^2 m_2^2$.[27]

Non-(V-A) local four-fermion interactions contributing to muon decay are generally expected in models with composite leptons. They are generated by constituent exchange.[28] The strength of these interactions is of the order of g^2/Λ_c^2 where g is an effective strong coupling constant and Λ_c is the compositeness scale. Assuming $g^2/4\pi \simeq 1$, muon decay provides for some types of couplings a lower bound of about 3 TeV on Λ_c.[15]

The framework for the description of muon decay we have discussed so far is not general enough to encompass all the possibilities one could encounter in extensions of the minimal standard model.

One of the reasons is that the neutrino states appearing in $SU(2)_L$ multiplets may be mixtures of mass-eigenstates.[29] The latter may include also heavy neutral fermions that cannot be produced in muon decay[30]. In the presence of lepton mixing e.g. the coupling of the W to leptons relevant to muon decay is of the form[31]

$$\begin{aligned} L &= (g/2\sqrt{2})\cos\theta^e_L\, \bar{e}\gamma_\lambda(1-\gamma_5)\sum_i (A^\nu_L)_{ei}\nu_i \\ &+ (g/2\sqrt{2})\sin\theta^e_R\, \bar{e}\gamma_\lambda(1+\gamma_5)\sum_i (F^\nu_R)_{ei}\nu_i + H.c. \\ &+ (e \to \mu) \quad . \end{aligned} \tag{29}$$

In Eq. (29) A^ν_L and F^ν_R are sub-matrices of the neutrino mixing matrices, responsible for the mixing of the light neutrinos; θ^e_L and θ^e_R are angles describing possible mixing of the electron with new heavy charged leptons. The right-handed current term in Eq. 29 appears when the right-handed components of some of the new leptons are in SU(2) doublets.

Even if neutrino mixing would be absent, there could be effects in muon-decay that are not described by the Hamiltonian (3). An example is the decay mode[32]

$$\mu^{+} \rightarrow e^{+} \nu_e \nu_\mu \tag{30}$$

which violates electron- and muon-number conservation.

The most general local nonderivative four-fermion interaction that allows for neutrino mixing, lepton-family-number and total lepton-number violation can be obtained from the Hamiltonian (3) by the replacements

$$\begin{aligned}
& g_{LL}^{V}\ \bar{e}\gamma_\lambda(1-\gamma_5)\nu_e\bar{\nu}_\mu\gamma^\lambda(1-\gamma_5)\mu \\
& \rightarrow \sum_{i,j}(g_{LL}^{V})_{ij}\ \bar{e}\gamma_\lambda(1-\gamma_5)n_i\bar{n}_j\gamma^\lambda(1-\gamma_5)\mu \quad , \\
& g_{RR}^{V}\ \bar{e}\gamma_\lambda(1+\gamma_5)\nu_e\bar{\nu}_\mu\gamma^\lambda(1+\gamma_5)\mu \\
& \rightarrow \sum_{i,j}(g_{RR}^{V})_{ij}\bar{e}\gamma_\lambda(1+\gamma_5)n_i{}'\bar{n}_j{}'\gamma^\lambda(1+\gamma_5)\mu \quad ,
\end{aligned} \tag{31}$$

and analogous replacements for all the other terms in (3) (Ref. 33). In Eq. 31 all left-handed neutrinos (i.e. in the case of Dirac neutrinos both the left-handed neutrinos and the left-handed antineutrinos) have been denoted by $(1-\gamma_5)n$ and similarly all right-handed neutrinos by $(1+\gamma_5)n'$.

The constraints on the general interaction have been analyzed in Ref. 33, assuming that the effects of the neutrino masses on the positron spectrum can be neglected. The positron spectrum can again be described by the 10 parameters $a, a', \ldots$, which are quadratic functions of the coupling constants of the interaction. One can form again six positive semidefinite quadratic forms, which are the same functions of the muon decay parameters $\rho, \delta, \ldots$ as in the case of the interaction (3). The quantity $Q_{LL}(= 1 - Q_{RR} - Q_{LR} - Q_{RL})$, for example, is now given by

$$Q_{LL} = \sum_{i,j} \left(|(g_{LL}^{V})_{ij}|^2 + \frac{1}{4}|(g_{LL}^{S})_{ij}|^2 \right) . \tag{32}$$

One has $Q_{LL} > 0.949$, as before.

Denoting n_{3L} the neutrino state $\nu_\pi = \sum_j c_j n_{jL}$ produced in $\pi^+ \to \mu^+\nu_\pi$ decay, the cross-section ratio S is given by

$$S \simeq \sum_i \left|(g^V_{LL})_{i3} + \frac{1}{2}(g^S_{LL})_{3i}\right|^2 . \tag{33}$$

From the experimental result (12) and from $Q_{LL} \leq 1$ one obtains

$$\sum_i \left|(g^V_{LL})_{i3} + \frac{1}{2}(g^S_{LL})_{3i}\right|^2 > 0.79 , \tag{34}$$

$$\sum_{\substack{i,j\\ j\neq 3}} \left|(g^V_{LL})_{ij} + \frac{1}{2}(g^S_{LL})_{3i}\right|^2 < 0.21 . \tag{35}$$

Thus the conclusion in the general case is[33] that the decay mode involving the neutrino ν_π produced in $\pi^+ \to \mu^+\nu_\pi$ decay dominates the muon decay rate. The contributions of the scalar and the vector terms in (34) cannot be separated.

There is some experimental information also concerning the second neutrino emitted in muon decay. This comes from an experiment[34] which measured the ratio $\Gamma(\mu^+ \to e^+\bar{\nu}_e\nu_\mu)/\Gamma(\mu^+ \to \text{all})$ using the reactions $\bar{\nu}p \to ne^+$ and $\nu_e d \to ppe^-$ induced by neutrinos from muon decay. The result is

$$R \equiv \Gamma(\mu^+ \to e^+\bar{\nu}_e\nu_\mu)/\Gamma(\mu^+ \to \text{all}) < 0.098 \qquad (90\%\ \text{c.l.}). \tag{36}$$

The good agreement of the measured $\nu_e d \to ppe^-$ cross section and the calculated one in the minimal standard model indicates that the total muon decay rate contains a substantial contribution from muon decay into a final state in which one of the neutrinos is the one accompanying the positron in beta-decay.

2.2 Radiative Muon Decay

The decay $\mu^+ \to e^+\nu_e\bar{\nu}_\mu\gamma$ (and $\mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma$) has a branching ratio of about 10^{-4} relative to nonradiative muon decay. The transition probability with the neutrinos unobserved depends on the same 10 parameters $a, a', \ldots$ as the transition probability for $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ (Ref. 35). Without considering observables involving positron or photon polarizations, from measurements of radiative muon decay one can determine two new combinations of the parameters: $\bar{\eta}$ (in unpolarized muon decay) and κ (in polarized muon decay).[36] In the minimal standard model $\bar{\eta} = \kappa = 0$. A recent direct measurement[37] yields $\bar{\eta} < 0.083$ (68% c.l.). A better limit $\bar{\eta} < 0.033$ (Ref. 38) follows from combining results from nonradiative muon decay.

A question of interest concerning the decay $\mu^+ \to e^+\nu_e\bar{\nu}_\mu\gamma$ is whether it could offer an easier way to probe possible time-reversal violation in the muon-decay interaction. One would expect a priori that this may be the case, since one can form T-odd correlations without including the positron spin vector (e.g. $\vec{\sigma}_\mu \cdot \vec{p}_e \times \vec{p}_\gamma$). An analysis[35] based on the Hamiltonian (3) shows however that (a) only parity-violating correlations can be nonvanishing (so that e.g. $\vec{\sigma}_\mu \cdot \vec{p}_e \times \vec{p}_\gamma$ does not appear), and (b) parity-conserving correlations either involve the transverse positron polarization, or the longitudinal positron polarization. Moreover, correlations involving the longitudinal polarizations (such as for example $\vec{\sigma}_e \cdot \vec{p}\vec{\sigma}_\mu \cdot \vec{p}_e \times \vec{p}_\gamma$) are proportional to the electron mass (and therefore suppressed, except possibly for low-energy positrons).

An analysis of $\mu \to e^+ + neutrinos + \gamma$ in the framework of an interaction which allows for neutrino mixing and nonconservation of lepton numbers has not been yet, to our knowledge, made.

3. CHARGED PION DECAYS

3.1 $\pi \to \ell\nu_\ell$

In the minimal standard model the effective interaction describing $\pi \to \ell\nu_\ell$ ($\ell = e, \mu$) decays is described by the V-A Hamiltonian[39]

$$H^{(\pi)}_{V-A} = (G_F/\sqrt{2})\, U_{ud}\, \bar{\nu}_\ell\gamma^\lambda(1-\gamma_5)\ell\; \bar{d}\gamma_\lambda(1-\gamma_5)u + H.c. \quad , \qquad (37)$$

where $U_{ud}(= \cos\theta_1)$ is an element of the quark mixing matrix. Only the $\bar{d}\gamma_\lambda\gamma_5 u$ part of (37) contributes to $\pi^+ \to \ell^+\nu_\ell$.

The decay rate due to (37) is given by

$$\Gamma(\pi \to \ell\nu_\ell) = (m_\pi/4\pi)(1-r_\ell^2)^2 m_\ell^2\, f_\pi^2\, U_{ud}^2\, (G_F/\sqrt{2})^2 , \qquad (38)$$

where $r = m_\ell / m_\pi$, m_ℓ is the mass of the charged lepton and f_π is defined by $< 0 \, |\bar{d}\gamma_\lambda \gamma_5 u| \, \pi^+(p) = i f_\pi p_\lambda ; f_\pi \simeq 132$ MeV (Ref. 40).

From the point of view of searches for new physics the quantity of interest is the ratio of the rates of the electronic and the muonic decay modes, in which the factor $f_\pi \, U_{ud}$ drops out. Including radiative corrections, the ratio

$$R \;\equiv\; \Gamma\Big((\pi \;\rightarrow\; e\nu) \;+\; (\pi \;\rightarrow\; e\nu\gamma)\Big) / \Gamma\Big((\pi \;\rightarrow\; \mu\nu) \;+\; (\pi \rightarrow\; \mu\nu\gamma)\Big) \qquad (39)$$

in the minimal standard model is predicted to be[41)]

$$R \;=\; 1.233 \times 10^{-4} \quad . \qquad (40)$$

The theoretical uncertainty in the prediction (40) is due to pion- structure dependent radiative corrections, not included in (40), which have been estimated to be $\lesssim$ 0.3% (Ref. 41).

Let us consider $\pi \rightarrow \ell\nu_\ell (\pi^+ \rightarrow \ell^+\nu_\ell$ for simplicity) in the framework of the Hamiltonian

$$H^{(\pi)} \;=\; \sum_{\alpha,\beta = L,R} \left(a^{(\ell)}_{\alpha\beta} \, \bar{\nu}_e \gamma_\lambda \Gamma_\alpha \ell \; \bar{d} \gamma^\lambda \Gamma_\beta u \;+\; A^{(\ell)}_{\alpha\beta} \bar{\nu}_e \Gamma'_\alpha e \bar{d} \Gamma_\beta u \right) \;+\; H.c. \qquad (41)$$

$$+ \; (e \;\rightarrow\; \mu) \quad ,$$

where, as before, $\Gamma'_R \;=\; \Gamma_L \;=\; 1 - \gamma_5$, $\Gamma'_L \;=\; \Gamma_R \;=\; 1 + \gamma_5$, and $a^{(\ell)}_{\alpha\beta}, A^{(\ell)}_{\alpha\beta}$ are constants. We shall assume that ν_e and ν_μ are mass-eigenstates, and neglect the effects of their possible masses.

The part of (41) containing the axial-vector and pseudoscalar quark currents is the most general interaction that can contribute to $\pi^+ \rightarrow \ell^+ \nu_\ell$. The $\pi^+ \rightarrow \ell^+\nu_\ell$ rate resulting from the interaction (41) is given by

$$\Gamma(\pi^+ \;\rightarrow\; \ell^+\nu_\ell) \;=\; (m_\pi / 4\pi)(1 \;-\; r_\ell^2)^2 \; m_\ell^2 \; f_\pi^2 \; Q^{(\ell)} \qquad (42)$$

$(\ell \;=\; e, \mu)$, where

$$Q^{(\ell)} \;=\; \sum_{\alpha = L,R} \left| \left(a^{(\ell)}_{\alpha L} \;-\; a^{(\ell)}_{\alpha R} \right) \;+\; \frac{m_\pi}{m_\ell} \; \frac{m_\pi}{m_u \;+\; m_d} \left(A^{(\ell)}_{\alpha L} \;-\; A^{(\ell)}_{\alpha R} \right) \right|^2 \quad ; \quad (43)$$

m_u and m_d are the masses of the u- and d-quarks.[42]

The ratio R (neglecting new physics contributions to the radiative corrections) can be written as

$$R = (1.233 \times 10^{-4})\,(Q^{(e)}/Q^{(\mu)}) \quad . \tag{44}$$

The present experimental value of R is[43,44]

$$R_{\mathrm{expt}} = (1.218 \pm 0.014) \times 10^{-4} \quad . \tag{45}$$

Analysis of the results of a new experiment measuring R is under way.[45] The result (45) implies for $Q^{(e)}/Q^{(\mu)} - 1$ $(= (R/R_{V-A}) - 1)$

$$|(Q^{(e)}/Q^{(\mu)}) - 1| < 0.034 \qquad (90\%\ \mathrm{c.l.}) \tag{46}$$

In the upper limit (45) we have included the theoretical uncertainty of $\sim 0.3\%$ in the calculation of the coefficient of $Q^{(e)}/Q^{(\mu)}$ in Eq. 44.

As seen from Eq. (43) the ratio R is sensitive to pseudoscalar couplings of the type for which $A^{(e)}_{\alpha\beta}/A^{(\mu)}_{\alpha\beta} \neq m_e/m_\mu$, and to new axial-vector interactions if $a^{(e)}_{\alpha\beta} \neq a^{(\mu)}_{\alpha\beta}$.

P-type couplings occur generally in models with charged Higgs bosons[46,47] and also in models involving leptoquarks.[48] Let us consider a pseudoscalar interaction involving a left-handed neutrino, added to the minimal standard model interaction.[48] From Eqs. 43 and 46 we obtain, assuming that there is no cancellation between the electronic and muonic terms,

$$|Re(\hat{A}^{(e)}_{LL} - \hat{A}^{(e)}_{LR})| < 5.2 \times 10^{-6} \tag{47}$$

$$|Re(\hat{A}^{(\mu)}_{LL} - \hat{A}^{(\mu)}_{LR})| < 1.1 \times 10^{-3} \tag{48}$$

where we have denoted $\hat{A}^{(\ell)}_{L\beta} = A^{(\ell)}_{L\beta}/(G_F/\sqrt{2})V_{ud}$. For comparison we note that the best limits on the absolute values of the coupling constants of scalar, tensor and $V+A$ charged current interactions, obtained from beta decay, are 10-20% G_F (Ref. 49). Limits on non-$(V-A)$ couplings involving the muon from charged-current processes other than pion decay are not likely to be better.[50]

An A_{R3}-term in (43) has no interference with the minimal standard model contribution. The limits on such couplings are therefore weaker. We obtain, barring cancellations,

$$|\bar{A}^{(e)}_{RR} - \bar{A}^{(e)}_{RL}| < 5.7 \cdot 10^{-5} \quad , \tag{49}$$

$$|\bar{A}^{(\mu)}_{RR} - \bar{A}^{(\mu)}_{RL}| < 1.2 \cdot 10^{-2} \quad , \tag{50}$$

($\bar{A}_{R3} = A_{R3}/(G_F/\sqrt{2})V_{ud}$).

The factor $Q^{(e)}/Q^{(\mu)}$ could deviate from one even in the absence of new interactions, due to mixing of e and μ with new heavy leptons. If, for example, the weak eigenstate left-handed electron mixes with a new sequential heavy charged lepton E, one has[51]

$$Q^{(e)}/Q^{(\mu)} = \cos^2\theta^e_L \quad , \tag{51}$$

where θ^e_L is the $e_L - E_L$ mixing angle. From Eq. (46) we obtain

$$|\sin\theta^e_L| < 0.18 \quad . \tag{52}$$

A systematic study of mixings of the usual light fermions with new heavy fermions was carried out in Ref. 52. The upper bound for $|sin\ \theta^e_L|$ when all light fermions are allowed to mix and all the pertinent data are taken into account is comparable to (52).

Further information on the Hamiltonian (41) can be obtained from the longitudinal polarization $P_L^{(\mu+)}$ of the muon in $\pi^+ \to \mu^+\nu_\mu$ (Refs. 47 and 53). For the interaction (41) it is given by (neglecting neutrino masses, as we do)

$$P_L^{(\mu+)} = (c_L^{(\mu)} - c_R^{(\mu)})/(c_L^{(\mu)} + c_R^{(\mu)}) \quad , \tag{53}$$

where

$$c_K^{(\mu)} = \left|(a^{(\mu)}_{KL} - a^{(\mu)}_{KR}) + \omega_\mu(A^{(\mu)}_{KL} - A^{(\mu)}_{KR})\right|^2 \tag{54}$$

($K = L, R$), and $\omega_\mu = m_\pi^2/m_\mu(m_u + m_d)$. In the absence of right handed neutrinos (as is the case in the minimal standard model) $P_L^{(\mu+)} = 1$.

From the limit (14) (noting that $P_L^{(\mu^+)} = h$) one has for $A^{(\mu)}_{Ri}$-terms added to the minimal standard model interaction

$$|A^{(\mu)}_{RR} - A^{(\mu)}_{RL}| < 2.5 \cdot 10^{-3} \quad . \tag{55}$$

For $a^{(\mu)}_{Ri}$-terms the limit is

$$\left|a^{(\mu)}_{RR} - a^{(\mu)}_{RL}\right| < 0.04 \quad . \tag{56}$$

The e-polarization in $\pi \to e\nu$ would also be of interest, but experimentally it is far less accessible.

So far we have neglected the effects of neutrino mass and mixing. If the neutrinos are massive and mix, the decay $\pi \to \ell\nu_\pi^{(\ell)}$ is an incoherent sum of the decays $\pi \to \ell\nu_i^{(\ell)}$ where $\nu_i^{(\ell)}$ are the mass eigenstates contained in $\nu_\pi^{(\ell)}$. If the kinematic effects of the masses of the neutrinos that can be produced in $\pi \to \ell\nu_\pi^\ell$ can be neglected, the mixing would have no effect on the $\pi \to \ell\nu_\pi^{(\ell)}$ rates, unless the neutrino state associated with ℓ includes heavy neutrinos that are energetically not allowed in $\pi \to \ell\nu_\pi^{(\ell)}$ (Ref. 30). If, for example, such heavy neutrinos are sequential leptons, the quantity $Q^{(e)}/Q^{(\mu)}$ would be given by

$$Q^{(e)}/Q^{(\mu)} = \cos^2\theta_L^{\nu_e}/\cos^2\theta_L^{\nu_\mu} \tag{57}$$

where $\cos^2\theta_L^{\nu_{(\ell)}} = \sum_i |(A_L^\nu)_{\ell i}|^2$ ($\ell = e,\mu$), and A_L^ν is the matrix in Eq. 29 (Ref. 52). Barring cancellations, the limit (46) implies

$$|\sin\theta_L^{\nu_\ell}| < 0.18 \quad (\ell = e,\mu) \quad . \tag{58}$$

The analysis given in Ref. 52 for the case when only the parameters $\sin\theta_L^{\nu_\ell}$ are included in the fit (but all the pertinent data are taken into account) yields $|\sin\theta_L^{\nu_e}| < 0.17$ and $|\sin\theta_L^{\nu_\mu}| < 0.049$.

If the masses of the neutrinos in ν_π^ℓ are not negligible, the charged lepton energy spectrum will contain additional peaks at energies given by m_{ν_i} and with heights proportional to $|U_{\ell i}|^2$ (U = neutrino mixing matrix).[54]

Peaks in the charged lepton spectrum have been searched for both in $\pi \to e\nu$ (Ref. 55) and $\pi \to \mu\nu$ (Ref. 56) decays. Limits on $|U_{ei}|^2$ and $|U_{\mu i}|^2$ have been obtained in the mass ranges $20\ MeV/c^2 < m_{\nu_i} < 130\ MeV/c^2$ and $1\ MeV/c^2 < m_{\nu_i} < 16\ MeV/c^2$, respectively. For $|U_{ei}|^2$ these bounds in the region $50\ MeV/c^2 < m_{\nu_i} < 120\ MeV/c^2$ are $\sim 10^{-7} - 3 \times 10^{-7}$, and for $|U_{\mu i}|^2$ in the region $4\ MeV/c^2 < m_{\nu_i} < 16\ MeV/c^2$ about $10^{-4} - 10^{-5}$. The decay $\pi \to \mu\nu$ provides also the best limit ("$m^{\prime\prime}_{\nu_\mu} < 0.25 MeV/c^2$, 90% c.l.) on the mass(es) of the dominant light neutrino(s) in $\nu_\pi^{(\mu)}$.

The masses of the neutrinos involved in $\nu_\pi^{(\ell)}$ affect also the ratio R (Refs. 54 and 58). Limits on $|U_{ei}|$ and $|U_{\mu i}|$ from R have been obtained in Ref. 59.

3.2 $\pi^+ \to \pi^o e^+ \nu_e$

In the minimal standard model the only contribution to the decay $\pi^+ \to \pi^o e^+ \nu_e$ comes from the $\bar{d}\gamma_\lambda u$-part of the Hamiltonian (37). The amplitude involves the matrix element

$$< \pi^o |\bar{d}\gamma_\lambda u| \pi^+ > = (p_+ + p_o)_\lambda f_+ + (p_+ - p_o)_\lambda f_- , \qquad (59)$$

where f_+ and f_- are functions of $(p_+ - p_o)^2$. Since $\bar{d}\gamma_\lambda u$ is a component of the isospin current, one has $f_- = 0$ and $f_+(0) = \sqrt{2}$ in the limit of isospin invariance. The $\pi^+ \to \pi^o e^+ \nu_e$ rate in this limit is predicted to be (see Ref. 60 and references quoted therein)

$$\Gamma(\pi^+ \to \pi^o e^+ \nu_e) = \frac{G_F^2 V_{ud}^2}{60\ \pi^3} \left(1 - \frac{\Delta}{2m_+}\right)^3 \Delta^5 F , \qquad (60)$$

where $\Delta = m_+ - m_o, m_+$ and m_o are the masses of π^+ and π^o, respectively, $\epsilon = m_e/\Delta$, and F is a function of ϵ and Δ. To compare theory and experiment, beta-decay information is used to eliminate the factor $G_F^2 V_{ud}^2$ and some of the radiative corrections. One obtains (see Ref. 60)

$$\Gamma(\pi^+ \to \pi^o e^+ \nu_e) = \frac{\ell n 2}{30\ ft'} \left(\frac{\Delta}{m_e}\right)^5 \left(1 - \frac{\Delta}{2m_+}\right)^3 F \left(1 + \delta_\pi^{(o)}\right) , \qquad (61)$$

where ft' is the ft-value for beta-decay corrected for outer radiative corrections, and $\delta_\pi^{(o)}$ is the outer radiative correction for pion decay.

Using the new value of the $\pi^+ - \pi^o$ mass-difference Δ = $(4.59366 \pm 0.00048) MeV$ (Ref. 61), Eq. (61) yields[62]

$$\Gamma(\pi^+ \rightarrow \pi^o e^+ \nu_e)_{th} = (0.3981 \pm 0.0008) s^{-1} \quad . \tag{62}$$

The experimental value is

$$\Gamma(\pi^+ \rightarrow \pi^o e^+ \nu_e)_{expt} = (0.394 \pm 0.015) s^{-1} \quad , \tag{63}$$

so that $\Gamma_{th} - \Gamma_{expt} = (1.03 \pm 3.77)\%$.

A large discrepancy between theory and experiment would require a reexamination of isospin breaking effects. From possible new physics $\pi^+ \rightarrow \pi^o e^+ \nu_e$ could receive a contribution from an interaction involving a tensor quark-current (an interaction with a scalar quark-current cannot contribute in the limit of G-parity conservation). Tensor-type semileptonic four-fermion couplings can be generated by spin-zero leptoquark exchange, and could also arise in composite models. $\Gamma(\pi^+ \rightarrow \pi^o e^+ \nu_e)_{th} - \Gamma(\pi^+ \rightarrow \pi^o e^+ \nu_e)_{expt}$ sets some limit on the strength of the tensor coupling. This bound has not been investigated yet.

3.3 $\pi \rightarrow e\nu\gamma$ and $\pi \rightarrow e\nu ee$

These decays are of interest for the information they can provide on the form factors describing the $\pi \rightarrow$ *vacuum* matrix elements of the product of the electromagnetic and weak currents.[39] They have been also considered (before the advent of gauge theories) as possible sources of information on time-reversal-violating interactions.

In the decay $\pi \rightarrow e\nu\gamma$ time-reversal-odd correlations necessarily involve positron or photon polarizations (see Ref. 64 and references quoted therein). In $\pi \rightarrow e\nu ee$ T-odd correlations can be formed using only momenta.[65]

In the minimal standard model T-violating effects in $\pi \rightarrow e\nu\gamma$ and $\pi \rightarrow e\nu ee$ will be negligible, since in this model T violation arises only in second order in the weak iterations.[66] In some extensions of the minimal standard model T-violating semileptonic interactions arise already in first order. For example, in left-right symmetric models[45] there is a first order strangeness conserving semileptonic T-violating interaction of strength $\lambda_\xi \simeq (g_R/g_L)\xi \sin(\alpha+\omega)$ (g_R, g_L are gauge coupling constants, ξ is the $W_L - W_R$ mixing angle; α, ω are CP-violating phases) relative to G_F (Ref. 67). $|\lambda_\xi|$ is constrained to be smaller than 2×10^{-3} by the experimental limit on the coefficient D of the T-odd correlation $< \vec{J} > \cdot \vec{p}_e \times \vec{p}_\nu$ in nuclear beta-decay, and (less reliably) to be smaller than $\sim 10^{-4}$ by ϵ'/ϵ and the electric dipole moment of the neutron.[67] Judging from the calculation in Ref. 64, this suggests that the transverse positron polarization in $\pi \to e\nu\gamma$ in these models is $\lesssim 10^{-4}$. Whether this is so, and whether there could be in some other models larger effects will require an analysis. A relevant issue for searches of T-odd correlations is the size of final state interaction effects. This, to my knowledge, in the decays $\pi \to e\nu\gamma$ and $\pi \to e\nu e e$ have not been yet investigated.

4. LEPTON FAMILY NUMBER NONCONSERVING PROCESSES

The present experimental upper limits on some of the lepton family number[6] nonconserving processes are shown in Table III. In the minimal standard model all these processes are forbidden. The underlying reason is the masslessness of the neutrinos and that the coupling of the Z^o and of the Higgs boson to the fermions are diagonal in family space.

In many extensions of the minimal standard model the conservation of lepton family numbers is broken and consequently $\mu \to e\gamma, \mu \to 3e$ etc. are expected to occur at some level.[75]

Let us consider the decay $\mu \to e\gamma$. If the neutrinos in the standard model are massive, conservation of lepton family numbers no longer holds. The amplitude is of the form (assuming $m_i \ll m_W$)[76]

$$\begin{aligned} M(\mu \to e\gamma) &\propto \sum_i U^*_{\mu i} U_{ei} \left(\frac{10}{3} - \frac{m_i^2}{m_W^2} + \ldots \right) \\ &= - \sum_i U^*_{\mu i} U_{ei} \frac{m_i^2}{m_W^2} \quad , \end{aligned} \tag{64}$$

where m_i is the mass of the i^{th} neutrino and U is the neutrino mixing matrix. The last equation in (64) is the consequence of the unitarity of U. The $\mu \to e\gamma$

branching ratio $B(\mu \to e\gamma) \equiv \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\nu\bar{\nu})$ corresponding to (64) is given by

$$B(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_i U^*_{\mu i} U_{ei} \frac{m_i^2}{m_W^2} \right|^2 . \qquad (65)$$

For the known three neutrinos $B(\mu \to e\gamma)$ is unobservably small ($< 10^{-20}$), due to the existing limits on their masses and mixings. Observable $B(\mu \to e\gamma)$ (as large as the present upper limit) is possible in the presence of a 4^{th} generation heavy neutrino in the standard model,[77] in left-right symmetric models with heavy right-handed neutrinos,[78,79] and in other models involving heavy neutrinos.[31]

In addition to heavy neutrinos there are other possible sources of lepton family number nonconservation that could lead to observable $B(\mu \to e\gamma)$. For example, $\mu \to e\gamma$ could proceed via a leptoquark and heavy quark loop, rather than a loop with the W and neutrinos.[80] In some classes of models with supersymmetry lepton family number violation can arise due to induced off-diagonal slepton mass terms;[81] in some of these models $\mu \to e\gamma$ could be in the observable range.[82]

The mechanisms of lepton family number violation mentioned in connection with $\mu \to e\gamma$ lead also to $\mu \to 3e, \mu \to e\gamma\gamma$ and to $\mu^- \to e^-$ conversion in nuclei. The decays $\mu \to 3e$ and $\mu \to e\gamma\gamma$ are expected to be generally less sensitive to lepton family number violation induced by loop diagrams than $\mu \to e\gamma$, since they occur in order higher than $\mu \to e\gamma$. This is true also for $\mu^- \to e^-$ conversion in nuclei, but coherent $\mu^- \to e^-$ conversion is enhanced roughly by a factor of A (for an isoscalar interaction) or $(Z - N)^2/A$ (for an isovector interaction.[83]

The decay $\mu \to 3e$ and $\mu^- \to e^-$ conversion are generally more sensitive than $\mu \to e\gamma$ to sources of lepton family number violation which gives rise to these processes at the tree level. Examples are the exchange of horizontal bosons and in the case of $\mu^- \to e^-$ conversion also the exchange of leptoquarks.

$\mu^- \to e^+$ conversion in nuclei violates both lepton family number conservation and total lepton number conservation. The lowest order diagram for this process is 4th order in the gauge and/or Higgs couplings. The predictions of the branching ratio $\Gamma(\mu^- \to e^+)/\Gamma(\mu^- \to \nu)$ in various extensions of the minimal standard model are far below presently observable levels (for a review see Ref. 75).

An interesting mechanism for some leptonic lepton family number nonconserving processes is present in an attractive version[84] of left-right symmetric models based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. The Higgs sector of this model contains the usual Higgs multiplet ϕ of quantum numbers $(T_L, T_R, Y) = (\frac{1}{2}, \frac{1}{2}, 0)$, which provides Dirac masses for the fermions. To break the gauge symmetry down to $U(1)_{e.m.}$ additional Higgs bosons are required. An attractive choice is to employ a Higgs triplet $\Delta_R(0,1,2)$. A nonzero vacuum expectation value for the neutral component of the Δ_R-field breaks $SU(2)_L \times SU(2)_R \times U(1)$ down to $SU(2)_L \times U(1)$, and the latter symmetry group is then reduced to $U(1)_{e.m.}$ by $<\phi>\neq 0$.

The Higgs triplet Δ_R couples to right-handed leptons. $<\Delta_R>\neq 0$ generates a Majorana mass for the right-handed neutrino, providing an explanation of the smallness of the masses of the usual neutrinos.

The existence of another Higgs triplet $\Delta_L(1,0,2)$, which couples to left-handed leptons, is required by left-right symmetry. The coupling of Δ_L to the first two lepton families is of the general form[85]

$$
\begin{aligned}
L_{\Delta_L} = \frac{1}{2} f_{ee}\, \bar{\nu}_e^c (1+\gamma_5)\nu_e\, \Delta_L^o - \frac{1}{\sqrt{2}} f_{ee}\, \bar{\nu}_e^c (1+\gamma_5) e\, \Delta_L^+ \\
- \frac{1}{2} f_{ee}\, \bar{e}^c (1+\gamma_5) e\, \Delta_L^{++} + H.c. + (e \to \mu).
\end{aligned} \tag{66}
$$

The Lagrangian (66) generates some lepton family nonconserving processes, including muonium to antimuonium conversion, $\mu^+ \to e^+\nu_\mu\bar{\nu}_e$ and $\mu \to 3e$.

The Lagrangian (66) gives rise to muonium $(M) \to$ antimuonium $(\bar{M})$ transitions (through the couplings $\mu^+ \to \mu^- \Delta^{++}$ and $e^- \Delta^{++} \to e^+$).[87] The effective $M \to \bar{M}$ Hamiltonian is given by

$$
H_\Delta^{M\bar{M}} = \frac{1}{4}\frac{f_{ee}f_{\mu\mu}^*}{m_{++}^2}\, \bar{e}^c (1-\gamma_5) e\, \bar{\mu}(1+\gamma_5)\mu^c + H.c. \quad . \tag{67}
$$

Using a Fierz transformation, and expressing the charge-conjugate fields in terms of the original ones, (67) can be rewritten into

$$H_{\Delta}^{M\bar{M}} = \frac{G_{++}}{\sqrt{2}}\,\bar{\mu}\gamma_\lambda(1-\gamma_5)e\,\bar{\mu}\gamma_\lambda(1-\gamma_5)e + H.c. \tag{68}$$

where

$$G_{++} = \sqrt{2}\, f_{ee}\, f_{\mu\mu}^{*}/8m_{++}^{2}, \tag{69}$$

and m_{++} is the mass of Δ_L^{++}.

The Hamiltonian (69) is of the same form as the Hamiltonian

$$H^{M\bar{M}} = \frac{G_{M\bar{M}}}{\sqrt{2}}\,\bar{\mu}\gamma_\lambda(1-\gamma_5)e\,\bar{\mu}\gamma_\lambda(1-\gamma_5)e \tag{70}$$

considered by Feinberg and Weinberg in their 1961 paper.[68] The probability $P(\overline{M})$ that the muon decays as μ^- rather than μ^+, if the system at $t=0$ is pure muonium, is given by[68]

$$P(\overline{M}) \simeq |\delta|^2/2\lambda^2 \simeq (2.5\times 10^{-5})\,(G_{M\bar{M}}/G_F)^2 \quad , \tag{70}$$

where λ is the muon decay rate and $\delta = 2 < \bar{M}|H^{M\bar{M}}|M >$. For the Hamiltonian (70) Feinberg and Weinberg find

$$\delta \simeq 2.1 \times 10^{-12}\,(G_{M\bar{M}}/G_F)\,eV \quad . \tag{72}$$

Thus for the Δ^{++}-mechanism δ is also given by Eq. (72), except for $G_{M\bar{M}} \to G_{++}$.

Two new experimental results on $M \to \bar{M}$ conversion have been presented at this Symposium: a TRIUMF experiment[70] yielded $P(\bar{M}) < 2\times 10^{-6}$ (90% c.l.), implying $G_{++} < 0.3G_F$ (the best present limit), and an experiment[71] at LAMPF obtained $P(\overline{M},4\tau) < 5.5\times 10^{-6}$ $P(\bar{M},4\tau)$ (probability to find an $\bar{M}$ atom in a time interval of 4 muon lifetimes), corresponding to $G_{++} < 0.5G_F$.

Another process mediated by (66), this time by the singly charged Δ_L is the decay $\mu^+ \to e^+\nu_\mu\bar{\nu}_e$ (see Eq. (30)) (through $\mu^+ \to \nu_\mu\Delta^+ \to \nu_\mu\bar{\nu}_e e^+$).[66] The effective Hamiltonian is

$$H^{\mu} = \frac{1}{2}\frac{f_{ee}\,f_{\mu\mu}^{*}}{m_{+}^{2}}\,\bar{\nu}_e^{c}(1-\gamma_5)e\,\bar{\mu}(1+\gamma_5)\nu_\mu^{c} + H.c. \quad , \tag{73}$$

or equivalently,

$$H^{\mu} = \frac{1}{4}\frac{f_{ee}f^{*}_{\mu\mu}}{m^2_{+}}\bar{\mu}\gamma_{\lambda}(1-\gamma_5)e\,\bar{\nu}_{\mu}\gamma^{\lambda}(1-\gamma_5)\nu_e + H.c. \quad . \tag{74}$$

Defining $G_{+} = \sqrt{2}f_{ee}f^{*}_{\mu\mu}/8m^2_{+}$ we obtain for the ratio $R \equiv \Gamma(\mu^{+} \to e^{+}\bar{\nu}_e\nu_{\mu})/\Gamma(\mu^{+} \to e^{+}\nu_e\bar{\nu}_{\mu})$

$$R = 4(G_{+}/G_F)^2 \quad . \tag{75}$$

The experimental limit (36) implies

$$G_{+} < 0.16G_F \quad . \tag{76}$$

The group associated with the $\nu_e e$ elastic scattering experiment at LAMPF (experiment 225) expects to reduce the present experimental error of 6% to ~ 1%.

Information on the interaction mediated by Δ^{+} among the same leptons is provided also by the experimental result[20]

$$\sigma(\nu_{\mu}e^{-} \to \mu^{-}\nu_e)/\sigma(\nu_{\mu}e^{-} \to \mu^{-}\nu_e) < 0.05 \tag{77}$$

but, to our knowledge, the implications for the Δ^{+}-mechanism, or for any other mechanisms, have not been analyzed yet.

Δ^{++}_L-exchange generates also the decay $\mu^{-} \to 3e$ (via $\mu^{+} \to e^{-}\Delta^{++} \to e^{-}e^{+}e^{+}$). The strength of this interaction (in the V-A form) is $G'_{++} = f_{ee}(f^{*}_{\mu\mu} - f^{*}_{ee})\sin\phi/m^2_{++}$, where ϕ is the $e-\mu$ mixing angle. G'_{++} is constrained to be small ($\lesssim 10^{-6}$) by the experimental limit on $B(\mu \to 3e)$.

The neutral component Δ^{0}_L mediates neutrino decay into three neutrinos.[89] Such decays, if fast enough, can provide the mechanism for intermediate mass neutrinos to evade the cosmological bounds.[89]

5. NEW PARTICLES IN CHARGED PION AND MUON DECAYS

In addition to decays which lead to final states containing only the known particles, the charged pions and the muon may have decay modes involving some new particles. Examples, which we have already considered in Sections 2 and 3, are the decays $\pi \to \ell N_k$, or $\mu \to e N_i N_j$ where N_i and/or N_j and N_k are new neutrinos. In this section we shall discuss briefly decays involving new particles other than neutrinos.

5.1 Supersymmetric Particles

Although the supersymmetric partners of the usual particles are expected to be heavy, the possibility that some of them are light is not ruled out.[90]

<u>5.1.1. $\mu^+ \to e^+ \widetilde{\nu}_e \widetilde{\overline{\nu}}_\mu$</u>. If the neutrinos ($\widetilde{\nu}$) are sufficiently light, the decay $\mu^+ \to e^+ \widetilde{\nu}_e \widetilde{\overline{\nu}}_\mu$ becomes possible. $\mu^+ \to e^+ \widetilde{\nu}_e \widetilde{\overline{\nu}}_\mu$ due to wino-exchange was studied in Ref. 91. If the masses of $\widetilde{\nu}_e$ and $\widetilde{\nu}_\mu$ can be neglected, the positron spectrum in (F1) can be parameterized in the same way as in ordinary muon decay. The spectrum parameters with $\mu^+ \to e^+ \widetilde{\nu}_e \widetilde{\overline{\nu}}_\mu$ added to the normal muon decay in $\mu \to e^+$ *missing neutrals* depend on $\epsilon = (m_W/m_w)^4$, where m_w is the wino mass. The most sensitive parameter turns out to be ξ($\xi \simeq 1 + 2\epsilon, \rho = \frac{3}{4}(1 + \frac{1}{2}\epsilon), \delta = \frac{3}{4}(1 - \frac{3}{2}\epsilon); (\xi\delta/\rho) - 1$ is quadratic in ϵ)[91]. The best limit on the wino mass is $m_w > 280$ GeV (90% c.l.), obtained from the experimental values of ρ and δ (Ref. 13). Approximately the same limit ($m_w > 270$ GeV (90% c.l.)) follows from the experimental value of the ξ parameter.[14]

<u>5.1.2. $\mu \to e\widetilde{\gamma}\widetilde{\gamma}$</u>. For light photinos ($\widetilde{\gamma}$) the decay $\mu \to e\widetilde{\gamma}\widetilde{\gamma}$ could occur. $\mu \to e\widetilde{\gamma}\widetilde{\gamma}$ mediated by scalar leptons was investigated in Ref. 92. The positron distribution in $\mu \to e +$ *missing neutrals* in the presence of $\mu \to e\widetilde{\gamma}\widetilde{\gamma}$ is different from that of normal muon decay. Limits on the effective slepton masses can be obtained from $P_\mu \xi\delta/\rho$, ξ', and from the transverse components of the positron polarization. Neglecting the photino mass, the lower bound on $P_\mu \xi\delta/\rho$ implies[92] $|M_R| \lesssim (350$ GeV$)$ (90% c.l.) for the effective mass (as defined in Ref. 92) of the sleptons corresponding to the right-handed components of the charged leptons. The limit from ξ' is weaker.[92] The experimental results on the transverse components of the positron polarization yield $|M_L/M_R| > (1.95 m_W)^2$ (90% c.l.) (Ref. 18).

5.2 Familons

Familons are geniune (massless) neutral Goldstone bosons, arising as a consequence of a supposed spontaneously broken continuous global family symmetry.[93,94] At low energies and at lowest order the coupling of a familon (as of other Goldstone bosons) to fermions is of the form[95]

$$L_F = \frac{1}{F} \varphi_f \bar{f}_1 [a(m_1 - m_2) + b(m_1 + m_2)\gamma_5] f_2 \quad , \tag{78}$$

where φ_f is the familon field, F is the scale at which the family symmetry is broken, m_1 and m_2 are, respectively, the masses of the fermions f_1 and f_2; a and b are constants.

Astrophysical and cosmological considerations lead for F to the bounds $10^9 GeV \lesssim F \lesssim 10^{12} GeV$ (Ref. 93). The most stringent laboratory limits come from searches for the decays $K^+ \to \pi^+ f$, $\mu \to ef$ and $\mu \to ef\gamma$.

For a Hamiltonian of the form (F1) with $a = 1, b = 0$ (or $a = 0, b = 1$) the branching ratio $B(\mu \to ef) \equiv \Gamma(\mu \to ef)/\Gamma(\mu \to \text{all})$ is given by[93]

$$B(\mu \to ef) = (2.5 \times 10^{14}\, (GeV^2))/F^2 \quad . \tag{79}$$

Assuming that the positron distribution in $\mu \to ef$ is isotropic (i.e. that either $a = 0$ or $b = 0$) the limit on $B(\mu \to ef)$ is

$$B(\mu \to ef) < 2.6 \times 10^{-6} \tag{80}$$

obtained from the results of the experiment on Ref. 15 measuring the positron momentum spectrum end point in polarized muon decay. The limit (80) implies (for $a = 1, b = 0$ or $b = 1, a = 0$)

$$F > 9.9 \times 10^9 GeV \quad . \tag{81}$$

If a and b are of comparable size, the positron distribution is not even approximately isotropic, and therefore the limit (81) does not apply. A limit on F for such a case is provided by the experimental limit[88]

$$B(\mu \to ef\gamma) < 1.1 \times 10^{-9} \qquad (90\%\ \text{c.l.}) \tag{82}$$

on the branching ratio $B(\mu \to ef\gamma) \equiv \Gamma(\mu^+ \to e^+ f\gamma)/\Gamma(\mu^+ \to \text{all})$ of $\mu \to ef\gamma$ decay. The double-differential distribution in the positron energy and the opening angle between the photon and the positron for $\mu \to ef\gamma$ was calculated in Ref. 96. This was used in the analysis leading to the limit (82). The implied lower bound for F is[68]

$$F > 3.1 \times 10^9 \; GeV \quad . \tag{83}$$

The present limit $B(K^+ \to \pi^+ f) < 3.8 \times 10^{-8}$ (Ref. 97) on the $K^+ \to \pi^+ f$ branching ratio $B(K^+ \to \pi^+ f) \equiv \Gamma(K^+ \to \pi^+ f)/\Gamma(K^+ \to \text{all})$ gives (assuming also here $a = 1, b = 0$ or $b = 1, a = 0$) a somewhat better limit on F than (81): the predicted branching ratio is $(2.75 \times 10^{13}) F^{-2}$ (Ref. 93), so that $F > 2.7 \times 10^{-10}$.

It is also of interest to search for the decay $\mu \to eX$ where X is a massive neutral particle.[98] Limits of the order of $3 \times 10^{-4} - 5 \times 10^{-5}$ for $B(\mu \to eX)$ have been set for $0 < m_X < 103.5$ MeV (exclusive the region 93.4 MeV $< m_X < 98.1$ MeV (Refs. 99 and 13)). Upper limits are available also for branching ratio of the decay $\mu^+ \to e^+\phi$ with a subsequent decay of the ϕ into e^+e^- (Ref. 100). In the mass region $2m_e < m_\phi < 100 MeV$ and for the lifetimes below $10^{-9}s$ limits on the branching ratio down to 2×10^{-12} have been set.

5.3 Light Higgs Bosons and Higgs-Like Particles

Light scalar or pseudoscalar particles (axions, light Higgs,...) could be produced in the decays $\pi^+ \to e\nu X$ or $\mu \to e\nu\bar{\nu}X$. The observed value of the $\pi^+ \to e^+\nu e^+e^-$ branching ratio[101] was instrumental in ruling out[102] the variant axion model.[103] The same decay can also be used to search for a light Higgs bosons (h), including the standard one. A recent analysis[104] finds that the available experimental information still allows a standard Higgs boson of any mass between 14 MeV and 1 GeV. The rate for $\pi^+ \to e^+\nu_e h$ assuming standard Higgs couplings, was calculated in Ref. 104. The recent data on $\pi^+ \to e^+\nu e^+e^-$ (Ref. 101) are likely to lead to limits on m_h in the range $2m_e < m_h < 80 MeV/c^2$ (Ref. 104).

A light Higgs boson could be searched for also in $\mu \to e\nu\bar{\nu}h$. However, the branching ratio for this decay was found to be smaller than the present sensitivities for muon decay of rare muon decay experiments.[104)]

For the inclusive process $\pi^+ \to e^+\nu X$ upper limits on the branching ratio of $4 \times 10^{-7} - 4 \times 10^{-6}$ have been set in the range $0 < m_X < 125 MeV/c^2$, assuming that X has a rest-frame life-time greater than 2 ns (Ref. 105). For life-times less than 2 ns the limits are weaker by up to an order of magnitude.[105)]

5.4 Majorons

A possible way to generate neutrino mass is to break spontaneously the global lepton number symmetry present in the minimal standard model. The resulting Goldstone bosons are called Majorons.

Two Majoron models have been discussed in the literature. In one of them[106)] the minimal standard model is extended by a right-handed singlet neutrino and by a singlet Higgs field which carries lepton number. The neutrinos acquire both a Dirac mass (m) (from the standard Higgs doublet) and a large Majorana mass (M) (from the singlet Higgs), so that the smallness of the masses of the usual neutrinos can in this model be understood. The majoron in this model has extremely weak couplings to the usual neutrinos ($\sim (m/M)^2$) and also to other fermions ($\sim G_F m_f m_\nu / 16\pi^2$).

In the second Majoron model only the Higgs sector of the minimal standard model is extended, adding a triplet Higgs field carrying lepton number.[107)] In this model the neutrinos can have almost equal mass: the masses do not follow the family hierarchy. In addition to the Majoron (χ) the model contains also a very light neutral Higgs boson (ϕ_h). The couplings of χ and ϕ_h to neutrinos is of the form[107,108)]

$$L = \frac{1}{2} \sum_{\ell,\ell'} g_{\ell\ell'} \nu_\ell (i\gamma_5 \chi + \phi_h) \nu_{\ell'} \qquad (81)$$

where $g_{\ell\ell'}$ is expected to be of the order of one, but could be smaller. The coupling of χ and ϕ_h to other fermions is much weaker.

In charged pion decays χ and ϕ_h can appear in $\pi \to e\nu\chi$ and $\pi \to e\nu\phi_h$ (χ and ϕ_h are emitted by ν).[108] In Ref. 105 the limit

$$\Gamma(\pi \to e\nu\chi(\phi_h))/\Gamma(\pi \to \mu\nu) < \text{[illegible]} \times 10^{-6} \tag{85}$$

has been set for these decays. This implies the limit

$$(g^2)_{ee} < 2 \times 10^{-4} \tag{86}$$

on the Majoron-neutrino coupling. In Eq. (86) g^2 is the square of the matrix whose elements are $g_{\ell\ell'}$ (see Ref. 108).

Majoron (or ϕ_h) emission affects also the ratio R in Eq. (44), since then one observes $\pi \to \ell L^o$ where L^o includes $\nu, \nu\chi$ and $\nu\phi_h$. The prediction for R (Eq. 44) is[108]

$$R = 1 + 157.5\,(g^2)_{ee} \quad . \tag{87}$$

The experimental value of R (Ref. 43) yields[105]

$$(g^2)_{ee} < 7.6 \times 10^{-5} \quad . \tag{88}$$

In the presence of Majorons the decay $\mu^+ \to e^+ +$ *missing neutrals* would include[109] $\mu^+ \to e^+\nu_e\bar{\nu}_\mu Y$ and $\mu^+ \to e^+ Y Y'$ where $Y = \chi, \phi_h, Y' = \chi, \phi_h$. These processes have been calculated in Ref. 109. Given the limit (87) and the limit $(g^2)_{\mu\mu} < 2.4 \times 10^{-4}$ (from $K \to \mu$ decays) their effect would likely be too small to be observable.

6. CONCLUSIONS

In this talk we discussed the decays of the charged pion and of the muon from the point of view of physics beyond the minimal standard model. As we have seen, there is a great variety of possible new physics that can be probed by them. Below we summarize the main features.

• The usual muon decay probes the existence of non-(V-A) interactions. These could be generated, for example, by new gauge bosons, Higgs bosons, or by mixing of the usual fermions with new heavy fermions. They could also arise in composite models. It is important to continue to improve the accuracy of measurements of the muon decay parameters.

• The experimental value of the ratio $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ provides extremely stringent bounds on some pseudoscalar interactions involving the electron. Such interactions could arise, for example, through leptoquark exchange. The polarization of the muon in $\pi \to \mu\nu$ is sensitive to right-handed currents involving the muon. For pseudoscalar type currents of this kind the present experimental value of the polarization sets the best limits. $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ is sensitive also to some new light particles. The experimental accuracy for $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ could be improved by about a factor of 3 before encountering the estimated uncertainties in the radiative corrections.

• The experimental accuracy for the $\pi^+ \to \pi^o e^+ \nu_e$ branching ratio could be improved by about an order of magnitude before reaching the level of the estimated uncertainties in the radiative corrections. The decay $\pi^+ \to \pi^o e^+ \nu_e$ is sensitive to standard model effects that violate CVC (quark mass effects, electromagnetic corrections), and perhaps also to some new physics (effective tensor interactions).

• We have considered briefly the lepton family number nonconserving processes $\mu \to e\gamma$, $\mu \to 3e$, $\mu^+ \to e^+ \nu_\mu \bar{\nu}_e$, $\mu^+ e^- \to \mu^- e^+$ and $\mu^- N \to e^- N$. These processes probe a broad range of possible sources of lepton-family-number violation. Their rates could be as large as the present limits. The information they provide is complementary to the information one can obtain, for example, from lepton family nonconserving kaon decays. It is important to improve the limits on their branching ratios by as much as possible.

The anomalous muon decay $\mu^+ \to e^+ \nu_e \nu_\mu$ and muonium $\to$ antimuonium conversion probe an attractive class of left-right symmetric models. They are mediated in these models by the exchange of triplet Higgs bosons. The rates of $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$ and muonium $\to$ antimuonium conversion due to this mechanism can be as large as the present limits.

• Searches for new light particles (massive neutrinos, light Higgs bosons, etc.) are of continuing interest.

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TABLE I. Experimental values of muon-decay parameters

Parameter	Experimental Value	Minimal Standard Model Value	Reference
τ_μ	$(2.19703 \pm 0.00004)\times 10^{-6}$s		12
ρ	0.7106 ± 0.0026	3/4	12
δ	0.7486 ± 0.0026(stat.) ± 0.0028 (syst.)	3/4	13
ξ	1.0050 ± 0.0088	1	14
$P_\mu \xi \delta/\rho$	> 0.99682 (90% c.l.)	1	15
ξ'	0.998 ± 0.045	1	16
ζ''	0.65 ± 0.36	1	17
α/A	0.015 ± 0.052	0	18
β/A	0.002 ± 0.018	0	18
α'/A	-0.047 ± 0.052	0	18
β'/A	0.017 ± 0.018	0	18

TABLE II. 90% confidence level limits on the coupling constants in the Hamiltonian (3) in units of $G_\mu/\sqrt{2}$ (from Ref. 13).

$\lvert g^S_{LL}\rvert$	< 0.918	$\lvert g^T_{RL}\rvert$	< 0.122
$\lvert g^S_{RR}\rvert$	< 0.066	$\lvert g^V_{RR}\rvert$	< 0.033
$\lvert g^S_{LR}\rvert$	< 0.125	$\lvert g^V_{LR}\rvert$	< 0.060
$\lvert g^S_{RL}\rvert$	< 0.89	$\lvert g^V_{RL}\rvert$	< 0.110
$\lvert g^T_{LR}\rvert$	< 0.036	$\lvert g^V_{LL}\rvert$	< 0.888

TABLE III. Experimental upper limits on some lepton family member nonconserving processes.

Process		90% c.l. upper limits on the branching ratio (for $\mu^+e^- \to \mu^-e^+$ on the probability $P(\overline{M})$, defined in the text)	Reference
$\mu \to e\gamma$		4.9×10^{-11}	68
$\mu \to e\gamma\gamma$		7.2×10^{-11}	68
$\mu \to eee$		1×10^{-12}	69
$\mu^+ \to e^+\bar{\nu}_e\nu_\mu$		0.098	34
$\mu^+e^- \to \mu^-e^+$		2×10^{-6}	70
$\mu^- \to e^-$	Ti	4.6×10^{-12}	72
	Pb	4.9×10^{-10}	72
	S	7×10^{-11}	73
	Cu	1.6×10^{-8}	74
$\mu^- \to e^+$	Ti	1.7×10^{-10}	72
	S	9×10^{-10}	73
	Cu	2.6×10^{-8}	74